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Velocity-derivative skewness in small Reynolds number, nearly isotropic turbulence

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Previous measurements in the moderate to small Reynolds number range of isotropic turbulence have all shown the skewness factor $S \equiv -\overline{(\partial u/\partial x)^3}/[\overline{(\partial u/\partial x)^2}]^{\frac{3}{2}}$ of the streamwise velocity derivative to increase with decreasing Reynolds number. This 'paradoxical' trend was found for $150 \geqslant R_\lambda \geqslant 4$. New data covering the range $4 \geqslant R_\lambda \geqslant 1$ show a maximum S for R_λ between 4 and 3 and a rapid decrease for $R_\lambda < 2$.

1. Introduction

One of the central characteristics of fully developed turbulent flows is the negative value of the skewness of the streamwise velocity derivative, customarily characterized by its negative

$$S \equiv -\frac{\overline{(\partial u/\partial x)^3}}{[\overline{(\partial u/\partial x)^2}]^{\frac{3}{2}}}. \tag{1}$$

An essential consequence of the nonlinearity of the Navier–Stokes equations, S represents, among other effects, the average rate of production of mean-square vorticity by vortex stretching (Taylor 1938), a process which is absent from most traditional laminar flows. Starting with Batchelor & Townsend (1948), a number of people have reported measurements of S in various turbulent flows corresponding to turbulent Reynolds numbers $R_\lambda \equiv (\overline{u^2})^{\frac{1}{2}}\lambda/\nu$ between approximately 5 and 10000. Here λ is the 'Taylor microscale', $\overline{u^2} \equiv \frac{1}{3}\overline{u_k u_k}$ is the average of the mean-square turbulent velocity components and ν is the kinematic viscosity.

Some typical values of S for $R_\lambda > 4$ measured by previous investigators are presented in figure 1. Starting at the large R_λ end, we see that S is almost 1·0 for the largest R_λ attained, decreases to approximately 0·35 for $R_\lambda \approx 200$, then monotonically increases again to 0·55 for $R_\lambda \approx 5$. Persistence of this upward trend indefinitely with decreasing R_λ would seem paradoxical since for small enough R_λ the nonlinear terms in the Navier–Stokes equations become negligible, so one would intuitively expect S to vanish as well. The present study has consisted of the measurement of S in nearly

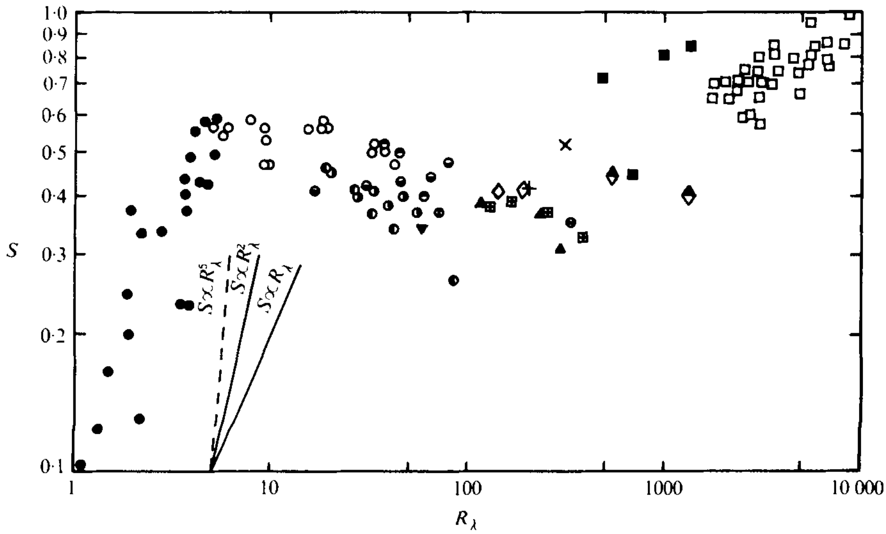

FIGURE 1. Measurements of the velocity-derivative skewness in various turbulent flows plotted *vs.* the turbulent Reynolds number (see table 1 for symbols).

Type of flow	Author(s)	Symbol
Nearly isotropic grid turbulence	Batchelor & Townsend (1949)	◑
	Stewart & Townsend (1951)	◐
	Mills *et al.* (1958)	⊖
	Frenkiel & Klebanoff (1971)	◓
	Kuo & Corrsin (1971)	⊗
	Betchov & Lorenzen (1974)	⊗
	Bennett & Corrsin (1978)	○
	Present data	●
Homogeneous shear flow	Tavoularis (1978)	◇
Duct flow	Comte-Bellot (1965)	▲
	Elena, Chauve & Dumas (1977)	▼
Mixing layers	Wyngaard & Tennekes (1970)	+
	Champagne, Pao & Wygnanski (1976)	×
Axisymmetric jet	Friehe, Van Atta & Gibson (1972)	⬘
	New measurements	⬙
Boundary layer	Ueda & Hinze (1975)	⊞
Atmosphere	Gibson, Stegen & Williams (1970)	■
	Wyngaard & Tennekes (1970)	□

TABLE 1

isotropic grid turbulence with R_λ as low as 1. Some theoretical predictions of the behaviour of S at small R_λ are also presented.

2. Some theoretical predictions

The particular velocity-derivative skewness defined in (1) is most simply related to the zero-separation third derivative of a particular triple correlation function (see, for example, Batchelor 1953, p. 119): in isotropic turbulence,

$$S(t) = -k'''(0,t)/[f''(0,t)]^{\frac{3}{2}}. \qquad (2)$$

Here $f(r)$ is the two-point velocity correlation function for components parallel to the line r joining the points and $k(r)$ is the two-point triple velocity correlation function for components parallel to the line joining the points. The primes denote differentiation with respect to r. The equivalent expression in terms of the three-dimensional spectral function $E(k,t)$ and energy transfer function $T(k,t)$ is

$$S(t) = \frac{3\times 30^{\frac{1}{2}}}{14}\int_0^\infty k^2T(k,t)\,dk\bigg/\left[\int_0^\infty k^2E(k,t)\,dk\right]^{\frac{3}{2}}. \tag{3}$$

The von Kármán–Howarth equation for the evolution of the double correlation in terms of the triple one can be used to express T in (3) in terms of E, giving S in terms of E alone:

$$S(t) = \tfrac{3}{7}\times 30^{\frac{1}{2}}\nu\frac{\displaystyle\int_0^\infty k^4E(k,t)\,dk}{\left[\displaystyle\int_0^\infty k^2E(k,t)\,dk\right]^{\frac{3}{2}}}+\tfrac{3}{14}\times 30^{\frac{1}{2}}\frac{\displaystyle\frac{d}{dt}\int_0^\infty k^2E(k,t)\,dk}{\left[\displaystyle\int_0^\infty k^2E(k,t)\,dk\right]^{\frac{3}{2}}}. \tag{4}$$

In the 'final period' of decay, the spectral transfer function becomes negligible and the spectral energy balance equation

$$\partial[E(k,t)]/\partial t+2\nu k^2E(k,t)\approx 0 \tag{5}$$

has the general solution

$$E(k,t) = C(k)\exp(-2\nu k^2t). \tag{6}$$

The form of $C(k)$ can be found by letting $k\rightarrow 0$ and making a plausible assumption. Batchelor & Townsend (1948; see also von Kármán & Howarth 1938) assumed in effect that all velocity cumulants have convergent integral moments to derive

$$\overline{u^2}\sim t^{-\frac{5}{2}}\quad\text{as}\quad t\rightarrow\infty \tag{7}$$

and

$$E(k,t)\sim k^4\exp(-2\nu k^2t). \tag{8}$$

On the other hand, Saffman (1967) assumed that all *vorticity* cumulants have convergent integral moments and concluded that

$$\overline{u^2}\sim t^{-\frac{3}{2}}\quad\text{as}\quad t\rightarrow\infty \tag{9}$$

and

$$E(k,t)\sim k^2\exp(-2\nu k^2t). \tag{10}$$

Experimental evidence seems to confirm (7) for grid-generated, nearly isotropic turbulence (Batchelor & Townsend 1948; Bennett & Corrsin 1978).

Assumptions (i.e. theories) are necessary in order to compute the asymptotic form of the spectral transfer function $T(k,t)$ in the final period of decay. Lin & Reid (1963) used Heisenberg's spectral theory and, with (3) and (8), concluded that

$$S\rightarrow\text{constant}\neq 0\quad\text{as}\quad t\rightarrow\infty\quad(R_\lambda\rightarrow 0). \tag{11}$$

It is of course doubtful that Heisenberg's assumptions are valid in the final period of decay. A possibly more realistic expression for $T(k,t)$ as $t\rightarrow\infty$ was derived by Deissler (1957) by discarding quadruple correlations in the balance equations for the triple correlations. The leading term of his expression was

$$T(k,t)\sim k^{12}t^{-\frac{3}{2}}\exp(-\tfrac{3}{2}\nu k^2t). \tag{12}$$

Substituting (12) and (8) into (3), we find that

$$S \sim t^{-\frac{15}{4}} \to 0 \quad \text{as} \quad t \to \infty. \tag{13}$$

If (10) were used instead of (8), the result would be

$$S \sim t^{-\frac{3}{2}} \to 0 \quad \text{as} \quad t \to \infty. \tag{14}$$

Since the asymptotic time dependence of the turbulence Reynolds number

$$R_\lambda \equiv u'\lambda/\nu$$

(u' = r.m.s. velocity, λ = Taylor microscale, ν = kinematic viscosity) is given by the von Kármán–Howarth/Batchelor–Townsend theory as

$$R_\lambda \sim t^{-\frac{3}{4}} \quad \text{as} \quad t \to \infty \tag{15}$$

and by the Saffman theory as

$$R_\lambda \sim t^{-\frac{1}{4}} \quad \text{as} \quad t \to \infty, \tag{16}$$

expressions (13) and (14) can be formulated in terms of R_λ as

$$S \sim R_\lambda^5 \quad \text{as} \quad R_\lambda \to 0 \tag{17}$$

and

$$S \sim R_\lambda^6 \quad \text{as} \quad R_\lambda \to 0. \tag{18}$$

The above analyses suggest that S decreases rapidly with R_λ at very small values of R_λ.

It should be pointed out parenthetically in this section that both the heat conduction equation

$$\partial\theta/\partial t = \mathscr{D}\,\partial^2\theta/\partial x^2 \quad (\mathscr{D} = \text{constant}) \tag{19}$$

and the Burgers equation

$$\frac{\partial\theta}{\partial t} + \theta\frac{\partial\theta}{\partial x} = K\frac{\partial^2\theta}{\partial x^2} \quad (K = \text{constant}) \tag{20}$$

give zero skewness for $\partial\theta/\partial x$ asymptotically for $t \to \infty$, independent of the initial skewness, for a general periodic initial distribution $\theta(x, 0)$.

3. Instrumentation and accuracies

The final measurements were conducted in the nearly isotropic turbulence behind a woven-wire square-mesh grid with mesh size $M = 1{\cdot}27$ mm and solidity 0·35. Four other grids of approximately the same solidity but with mesh sizes equal to 25·4 mm, 6·4 mm, 4·2 mm and 3·2 mm, generating turbulence with R_λ between 40 and 5, were also used in the earlier phases of this investigation (see Bennett & Corrsin 1978). Attempts to reduce R_λ further by decreasing the mean speed $\overline{U}$ through the $M = 3{\cdot}2$ mm grid failed; the wind-tunnel flow became erratic for speeds less than about 4 m/s. Therefore a finer grid was made, and reliable measurements of S in turbulence with R_λ as low as 1 were possible with the new grid.

The streamwise velocity fluctuation was measured with a hot wire of diameter 5 μm and length 1·2 mm, operated by a DISA 55D01 constant-temperature anemometer. The anemometer output was filtered with a Krohn–Hite model 330-M ultra-low-frequency band-pass filter and then digitized and processed in real time with the help of a PDP 11/40 digital mini-computer. The temporal derivative of the velocity

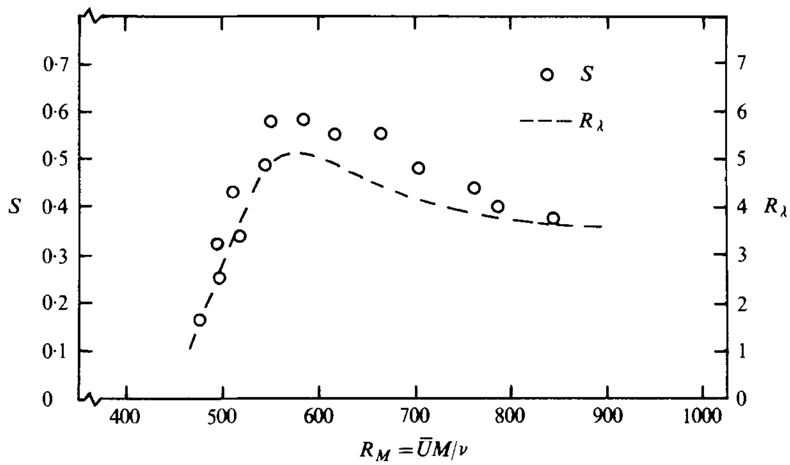

FIGURE 2. Measured values of the velocity-derivative skewness S and the turbulent Reynolds number R_λ for different grid Reynolds numbers (M = 1·27 mm).

was first computed from the difference between successive values of the digital time history and then transformed into the streamwise spatial derivative through use of Taylor's approximation. The sampling frequency was approximately four times the frequency associated with mean convection of the Kolmogorov microscale past the probe, viz. $4(\bar{U}/2\pi\eta)$, while the hot-wire length was nearly equal to or slightly smaller than the Kolmogorov microscale.

The instantaneous apparent velocity u_{ap} was the sum of the turbulent velocity u, the free-stream velocity fluctuation u_f (due possibly to fluctuations in the speed of the electric motor and flow irregularities in the settling chamber of the wind tunnel) and the electronic noise n. Assuming that these three terms were pairwise statistically independent, one finds that

$$\overline{u^2} = \overline{u_{ap}^2} - \overline{u_f^2} - \overline{n^2} \tag{21}$$

and

$$S = \overline{(\dot{u}_{ap}^3} - \overline{\dot{u}_f^3} - \overline{\dot{n}^3})/(\overline{\dot{u}_{ap}^2} - \overline{\dot{u}_f^2} - \overline{\dot{n}^2})^{\frac{3}{2}}, \tag{22}$$

where the dots indicate temporal derivatives.

The statistical properties of the sum $u_f + n$ were measured independently of u by replacing the grid with an empty frame; in this way proper corrections were applied for the recovery of $\overline{u^2}$ and S from the raw signal statistics. In addition, band-pass filtering was used to maximize the ratio $\overline{u^2}/\overline{u_{ap}^2}$. The high frequency setting f_H of the filter was approximately twice the Kolmogorov frequency $\bar{U}/2\pi\eta$, while the low frequency setting f_L was selected by trial and error as the value which reduced $\overline{u_f^2}$ by the maximum possible amount without appreciably affecting $\overline{u^2}$. Fortunately, most of the 'energy' of u_f was concentrated at much lower frequencies than the inverse of the integral time scale of u, so that in most cases the ratio $\overline{u^2}/\overline{u_{ap}^2}$ was found to be nearly equal to 1 for f_L in the range 1–5 Hz.

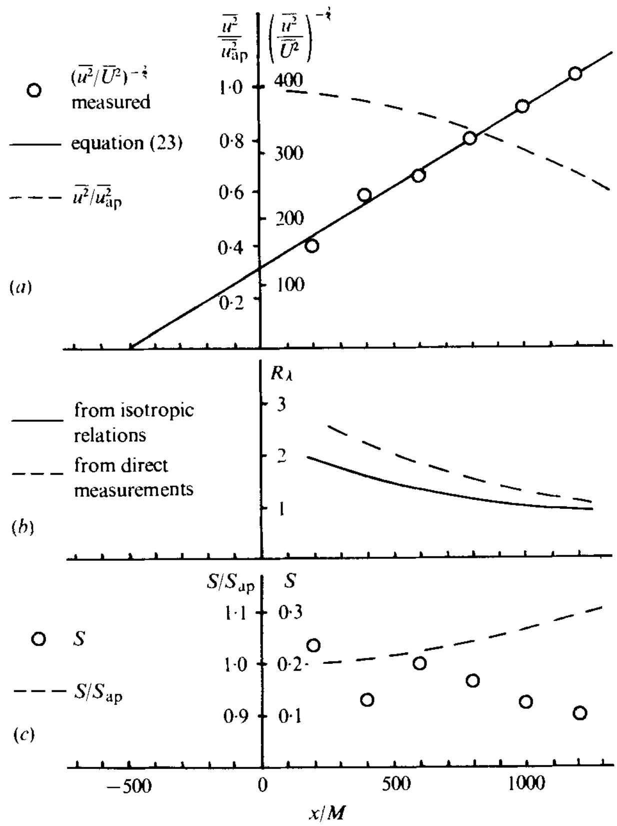

FIGURE 3. Measurements of the downstream development of (a) the turbulent kinetic energy, (b) the Taylor microscale and (c) the velocity-derivative skewness $S_{ap} = \overline{\dot{u}^3_{ap}}/(\overline{\dot{u}^2_{ap}})^{\frac{3}{2}}$. $R_M = 475$.

4. Results

Figure 2 shows the variation of the values of S and R_λ measured at a fixed position $x/M = 800$ behind the $M = 1{\cdot}27$ mm grid for different values of the grid Reynolds number $R_M = \bar{U}M/\nu$. Neither S nor R_λ shows a monotonic trend with decreasing R_M, but they both have a maximum at $R_M \approx 575$ and then rapidly decrease to the values $S \approx 0{\cdot}15$ and $R_\lambda \approx 1{\cdot}5$ at $R_M \approx 475$. The similarity of the behaviour of S and R_λ over the entire range of R_M is remarkable. It is worth mentioning that the random character of the hot-wire signal could still be observed on the oscilloscope at $R_M \approx 450$, but as R_M approached 400 it became clear that the flow ceased to be turbulent.

The downstream variation of $\overline{u^2}$, R_λ and S when $R_M \approx 475$ is shown in figure 3. An equation of the form

$$\overline{u^2} \propto (x/M + 500)^{-\frac{5}{2}}, \tag{23}$$

consistent with the theory of the final period of decay of Batchelor & Townsend (1948) and the data of Bennett & Corrsin (1978), was successfully fitted to the experimental values of $\overline{u^2}$, as seen in figure 3(a). The ratio $\overline{u^2}/\overline{u^2_{ap}}$, also shown in figure 3(a), varied between 1 and 0·7. Figure 3(b) is a plot of the values of R_λ computed with the aid of isotropic relations derived from (23) (solid line) and the values of R_λ obtained from direct measurements of $\overline{u^2}$ and λ (dashed line). The difference between the two curves must be attributed primarily to the lack of isotropy and secondly to experimental errors. Finally, figure 3 (*c*) shows that S decreases with downstream distance, following a similar decrease in R_λ. The correction due to systematic experimental errors in the measurement of S, as shown in the same figure, was at most of the order of 10 %.

The new experimental pairs of values of S and R_λ cover the range of R_λ between 5 and 1 in figure 1. The downward trend of S with decreasing R_λ is clear. The experimental scatter does not permit accurate determination of a relation between S and R_λ, but it appears that a power law $S \propto R_\lambda^2$ can be fitted very roughly to the data over the limited range $1 < R_\lambda < 2$.

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